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# OCCURRENCE OF THERMOCAPILLARY CONVECTION IN A CYLINDRICAL LAYER WITH DIFFERENT METHODS OF HEATING

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In the absence of body forces, perturbations of the equilibrium state of a nonuniformly heated fluid are governed by the intensity of the thermocapillary forces which arise as a result of the temperature dependence of surface tension. If the equilibrium temperature gradient is large enough, then a change in surface tension will lead to loss of stability of the equilibrium state – the occurrence of thermocapillary convection.

The studies [1-3] examined the conditions for the onset of convection in a fluid during heating of the solid or free boundary without allowance for the deformation of the free surface. Andreev et al. [4] studied the stability of the equilibrium of a fluid cylinder and cylindrical and plane layers heated by internal sources. The free surface was assumed to have been deformable in these cases. It was shown that allowance for the deformation of the boundary introduces a new factor which influences the stability of the equilibrium state. In this case, there is not only a decrease in stability, but there is a qualitative change in the neutral curve.

In the present investigation, we study the stability of a cylindrical layer with a deformable free surface in the case when the solid cylinder is also heated by internal sources. Formulas are obtained for the critical Marangoni numbers. It is shown that, as in [4], allowance for the deformation of the free boundary leads to discontinuities on the neutral curve. In the case of the heating of the solid surface, the curve of critical Marangoni numbers may have two points of discontinuity. Whether it does or not depends on the Weber number. Also, with heating by internal heat sources for azimuthal perturbations (m = 1), allowance for deformation of the free surface leads to an increase in stability.

1. We will examine a cylindrical layer of a viscous heat-conducting fluid bounded by a solid internal surface and free external surface. Gravitational forces are absent. We introduce a cylindrical coordinate system with the z axis directed along the generatrix of the cylinder. The equations of the solid and free boundaries $r = r_0$ and $r = r_1$, respectively. The change in surface tension as a function of temperature is described by the formula $\sigma = \sigma_0 - \varkappa(\theta - \theta_0)$.

Let the fluid contain permanent internal heat sources of intensity q, and let a constant temperature $\theta_1$ be assigned for the solid boundary. Then the equilibrium state is written as

$$u = v = w = 0,\ p = \text{const},$$
$$\theta(r) = -\frac{q}{4\chi}\left[r^2 - r_1^2 + \left(r_1^2 - r_0^2\right)\frac{\ln(r/r_1)}{\ln(r_0/r_1)}\right] + \theta_1\frac{\ln(r/r_1)}{\ln(r_0/r_1)}, \tag{1.1}$$

where u, v, and w are components of the velocity vector; p is pressure; θ is temperature.

We choose the quantities $r_1$, $r_1^2/\nu$, $\nu/r_1$, $\rho\nu^2/r_1^2$, $\rho\nu^2/\varkappa r_1$ as the characteristic scales of length, time, velocity, pressure, and temperature ($\nu$ and $\chi$ are kinematic viscosity and diffusivity and $\rho$ is density). After conversion to dimensionless form, the expression for temperature has the form

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$$\theta_0(\xi) = \mathrm{Ma}_q \mathrm{Pr}^{-1}[1 - \xi^2 - (1 - d^2)\ln\xi/\ln d]/2 - \mathrm{Ma}\mathrm{Pr}^{-1}\ln\xi. \tag{1.2}$$

Here, $\xi = r/r_1$; $d = r_0/r_1$; $\Pr = \nu/\chi$ is the Prandtl number; $\mathrm{Ma}_q = q\varkappa r_1^3/2\rho\nu\chi^2$, $\mathrm{Ma} = \theta_1\varkappa r_1/\rho\nu\chi\ln(r_1/r_0)$ is the Marangoni number.

Equations to describe small perturbations of arbitrary thermocapillary motion in cylindrical coordinates were obtained in [5]. Assuming that these disturbances are independent of $\varphi$, $\eta = z/r_1$, and $\tau = \nu t/r_1^2$ in accordance with the law $\exp[i(m\varphi + \alpha z) - i\alpha C\tau]$, we write the amplitude equations for the equilibrium state (1.2)

$$aU + P' = \left[\frac{1}{\xi}(\xi U)'\right]' - \frac{2im}{\xi^2}V; \tag{1.3}$$

$$aV + \frac{im}{\xi}P = \left[\frac{1}{\xi}(\xi V)'\right]' + \frac{2im}{\xi^2}U; \tag{1.4}$$

$$aW + i\alpha P = \frac{1}{\xi}(\xi W')'; \tag{1.5}$$

$$(\xi U)' + imV + i\alpha\xi W = 0; \tag{1.6}$$

$$bT - \Pr\theta_0' = \frac{1}{\xi}(\xi T')', \tag{1.7}$$

$$d < \xi < 1,\ a = \alpha^2 + m^2/\xi^2 - i\alpha C,\ b = \alpha^2 + m^2/\xi^2 - i\alpha\Pr C;$$

while the conditions on the solid boundary ($\xi = d$)

$$U = V = W = T = 0, \tag{1.8}$$

and the conditions on the free surface ($\xi = 1$)

$$\begin{gathered} V' - V + imU = -im\left(T + \theta_0' R\right), \quad i\alpha U + W' = \\ = -i\alpha\left(T + \theta_0' R\right), \quad -i\alpha CR = U, \\ -P + 2U' = \mathrm{We}_0(1 - \alpha^2 - m^2)R - \left(T + \theta_0' R\right), \quad T' + \mathrm{Bi}\,T + \\ + \left(\theta_0'' + \mathrm{Bi}\,\theta_0'\right)R = 0. \end{gathered} \tag{1.9}$$

In (1.3)-(1.9), U, V, W, P, and T are perturbations of the components of the vector of velocity, pressure, and temperature; R is the deviation of the boundary along a normal from its undisturbed state $r = r_1$; $\mathrm{We}_0 = r_1\sigma_0/\rho\nu^2$ is the Weber number; $\alpha$ is the wave number along the z axis; m is the spectral mode with respect to the angle $\varphi$; $C = C_r + iC_i$ is a complex decrement; $\mathrm{Bi} = \beta r_1/\lambda$ is the Biot number; $\lambda$ and $\beta$ are thermal conductivity and interphase heat-transfer coefficient.

We will henceforth examine only monotonic perturbations. In this case, the stability boundary is determined by the values C = 0 in (1.3)-(1.9). The conditions of existence of a nontrivial solution to the problem make it possible to find the critical Marangoni numbers at which the equilibrium state becomes unstable.

2. We seek the solution of (1.3)-(1.9) in the form $(U, V, W, P) = [T(1) + \theta_0'(1)R]\cdot(\varphi(\xi), \psi(\xi), g(\xi), f(\xi))$. In this case, the problem for the functions $\varphi$, $\psi$, g, and f is separate. Its solution was obtained [4]. We use (1.7) to find the function T and then, with allowance for (1.8)-(1.9) we obtain a relation linking the critical Marangoni numbers. We will present the final expressions for the three main cases.

For axisymmetric perturbations (m = 0)

$$\begin{gathered} \mathrm{Ma}\,\{\alpha^2(1 - \alpha^2)G_0(\alpha, d) + A_0(\alpha, d)\Pr^{-1}\mathrm{We}_0^{-1}[l_1 - \alpha l_2]\} + \\ + \mathrm{Ma}_q\,\{\alpha^2(1 - \alpha^2)G_{0q}(\alpha, d) + A_0(\alpha, d)\Pr^{-1}\mathrm{We}_0^{-1}\times \\ \times[\alpha l_2(1 + (1 - d^2)/2\ln d) + l_2(1 - (1 - d^2)/2\ln d)]\} = (1 - \alpha^2)(\mathrm{Bi}\,l_1 - \alpha l_2), \end{gathered} \tag{2.1}$$

where

$$l_1 = I_0(\alpha d)K_0(\alpha) - I_0(\alpha)K_0(\alpha d),\ l_2 = I_0(\alpha d)K_1(\alpha) + I_1(\alpha)K_0(\alpha d),$$
$$A_0(\alpha, d) = 1 - \alpha^2 + 2\alpha^2 C_3[I_1(\alpha) - \alpha I_0(\alpha)] + 2\alpha^2 C_4[K_1(\alpha) + \alpha K_0(\alpha)],$$

$$G_0(\alpha, d) = \int_d^1 \varphi_0(\tau)\,[K_0(\alpha\tau)\,I_0(\alpha d) - I_0(\alpha\tau)\,K_0(\alpha d)]\,d\tau,$$

$$G_{0q}(\alpha, d) = \int_d^1 (\tau^2 + (1-d^2)/2\ln d)\,\varphi_0(\tau)\,[K_0(\alpha\tau)\,I_0(\alpha d) - I_0(\alpha\tau)\,K_0(\alpha d)]\,d\tau,$$

$$\varphi_0(\xi) = \frac{C_1}{2\alpha}\,\xi I_0(\alpha\xi) - \frac{C_2}{2\alpha}\,\xi K_0(\alpha\xi) + C_3 I_1(\alpha\xi) + C_4 K_1(\alpha\xi),$$

$$C_1 = (1 - C_2 K_1(\alpha))/I_1(\alpha),\ C_2 = [-I_1(\alpha d)k_1 - I_0(\alpha d)k_2 + I_0(\alpha)/\alpha d^2]/\Delta,$$

$$C_3 = [-K_2(\alpha d)k_3 - K_1(\alpha d)k_4 + K_1(\alpha)/\alpha]/2\alpha\Delta,$$

$$C_4 = [-I_2(\alpha d)k_3 + I_1(\alpha d)k_4 - I_1(\alpha)/\alpha]/2\alpha\Delta,$$

$$\Delta = k_1^2 + 1/\alpha^2 d^2 - k_2[K_0(\alpha d)\,I_1(\alpha) + I_0(\alpha d)\,K_1(\alpha)],$$

$$k_1 = K_1(\alpha d)I_1(\alpha) - I_1(\alpha d)K_1(\alpha),\ k_2 = K_2(\alpha d)I_1(\alpha) + I_2(\alpha d)K_1(\alpha),$$

$$k_3 = I_0(\alpha)K_0(\alpha d) - K_0(\alpha)I_0(\alpha d),\ k_4 = K_0(\alpha)I_1(\alpha d) + I_0(\alpha)K_1(\alpha d).$$

The integrals $G_0$ and $G_{0q}$ can be expressed explicitly through modified Bessel functions. These functions are omitted here due to their awkwardness.

In the case of azimuthal perturbations ($\alpha = 0$) at $m \geq 2$

$$\begin{aligned}&\mathrm{Ma}\,\{(1-m^2)\,G(d) + A(d)\,\mathrm{Pr}^{-1}\,\mathrm{We}_0^{-1}\left[m\left(d^m + d^{-m}\right) - \left(d^m - d^{-m}\right)\right]\} + \\ &+ \mathrm{Ma}_q\,\{(1-m^2)\,G_q(d) - A(d)\,\mathrm{Pr}^{-1}\,\mathrm{We}_0^{-1}\left[m\left(d^m + d^{-m}\right)(1 + (1-d^2)/2\ln d) + \right. \\ &\left. + \left(d^m - d^{-m}\right)(1 - (1-d^2)/2\ln d)\right]\} = (1-m^2)\left[m\left(d^m + d^{-m}\right) - \mathrm{Bi}\left(d^m - d^{-m}\right)\right],\end{aligned} \quad (2.2)$$

where

$$A(d) = 2(1-m^2)(C_3 + C_6) + m^2 + 1, \quad G(d) = \int_d^1 \varphi(\tau)\left[d^m\tau^{-m} - d^{-m}\tau^m\right]d\tau,$$

$$G_q(d) = \int_d^1 (\tau^2 + (1-d^2)/2\ln d)\,\varphi(\tau)\left[d^m\tau^{-m} - d^{-m}\tau^m\right]d\tau,$$

$$\varphi(\xi) = (-mC_1\xi^{m+1} + mC_2\xi^{-m+1} + C_3\xi^{-m-1} + C_4\xi^{m-1})/2,$$

$$C_1 = [1 - d^{-2m} - m(1 - d^{-2})]/2\Delta,\ C_2 = -[1 - d^{2m} + m(1 - d^{-2})]/2\Delta,$$

$$C_3 = m(C_1 - 0.5),\ C_4 = -m(C_2 - 0.5),\ \Delta = d^{2m} - d^{-2m} - m(d^2 - d^{-2}).$$

With $\alpha = 0$ and $m = 1$, the problem for azimuthal perturbations becomes degenerate and the solution takes the form $U = V = P = 0$, $T = C_4(\xi - d^2/\xi)$, $C_4$ = const – which corresponds to the displacement of a cylindrical layer as an integral whole. If we put $\mathrm{We}_0 = \infty$ in (1.9), then the solution of this problem is nontrivial and the Marangoni number is determined by Eq. (2.1). In this case,

$$\varphi(\xi) = [-C_1\xi^2 + (C_2 - C_4/2) + C_3\xi^{-2} + C_4\ln\xi]/2,$$

where

$$C_1 = 0.5 + C_3,\ C_2 = (1 + d^2)/2 + C_3(d^2 + d^{-2}),$$

$$C_4 = d^2 + 2C_3(d^2 + d^{-2}),$$

$$C_3 = (1 - d^2 + 2d^2\ln d)/[2(d^2 - d^{-2}) - 4(d^2 + d^{-2})\ln d].$$

In the general case ($\alpha \neq 0$, $m \neq 0$) we have

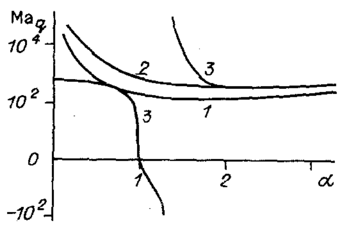


Fig. 1

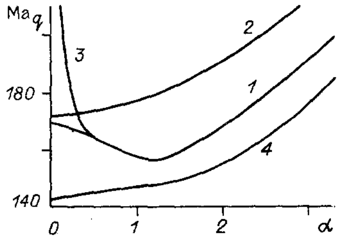


Fig. 2

$$f(\xi) = C_1 I_m(\alpha\xi) + C_2 K_m(\alpha\xi),\ \varphi(\xi) = [C_1 \xi I_m(\alpha\xi) +$$
$$+ C_2 \xi K_m(\alpha\xi) + C_3 I_{m+1}(\alpha\xi) + C_4 K_{m+1}(\alpha\xi) + C_5 I_{m-1}(\alpha\xi) + C_6 K_{m-1}(\alpha\xi)]/2,$$
$$\psi(\xi) = -i[C_3 I_{m+1}(\alpha\xi) + C_4 K_{m+1}(\alpha\xi) - C_5 I_{m-1}(\alpha\xi) - C_6 K_{m-1}(\alpha\xi)]/2,$$

the constants $C_1, \ldots, C_6$ being found from the boundary conditions $\varphi(d) = \psi(d) = \varphi'(d) = \varphi(1) = 0$, $\psi'(1) + \psi(1) + im = 0$, $C_1 I_m'(\alpha) + C_2 K_m'(\alpha) + 2im\psi(1) = -(\alpha^2 + m^2)$. Let us now present the final expression

$$\begin{gathered}\mathrm{Ma}\,\{(1-\alpha^2-m^2)\,G_m(\alpha, d) + A_m(\alpha, d)\,\mathrm{Pr}^{-1}\,\mathrm{We}_0^{-1}\,(l_2 - l_1)\} + \\ + \mathrm{Ma}_q\,\{(1-\alpha^2-m^2)\,G_{mq}(\alpha, d) - A_m(\alpha, d)\,\mathrm{Pr}^{-1}\,\mathrm{We}_0^{-1}\,[l_1(1 - \\ -(1-d^2)/2\ln d) + l_2(1 + (1-d^2)/2\ln d)]\} = (1-\alpha^2-m^2)(l_2 - \mathrm{Bi}\,l_1),\end{gathered} \tag{2.3}$$

where $l_1 = I_m(\alpha d)K_m(\alpha) - I_m(\alpha)K_m(\alpha d)$, $l_2 = I_m'(\alpha)K_m(\alpha d) - K_m'(\alpha)I_m(\alpha d)$,

$$G_m(\alpha, d) = \int_d^1 \varphi(\tau)\,[K_m(\alpha\tau)\,I_m(\alpha d) - I_m(\alpha\tau)\,K_m(\alpha d)]\,d\tau,$$
$$G_{mq}(\alpha, d) = \int_d^1 (\tau^2 + (1-d^2)/2\ln d)\,\varphi(\tau)\,[K_m(\alpha\tau)\,I_m(\alpha d) -$$
$$- I_m(\alpha\tau)\,K_m(\alpha d)]\,d\tau, \quad A_m(\alpha, d) = -f(1) + 2\varphi'(1) + 1.$$

Since the Weber and Prandtl numbers exist only as products in each of the relations derived here, we can reduce the number of determining parameters of the problem by introducing the modified Weber number $\mathrm{We} = \mathrm{We}_0\mathrm{Pr} = r_1\sigma_0/\rho\nu\chi$.

3. Let us examine the case Ma = 0, corresponding to heating by internal heat sources with an ideally conducting solid surface.

Figure 1 shows the graph of $\mathrm{Ma}_q$ as a function of α plotted from (2.1) for d = 0.1. Curves 1 and 2 correspond to the case of a nondeformable free boundary (We = ∞). For Bi = 0, the critical Marangoni number $\mathrm{Ma}_{q*} = 185.7$ (curve 1) at α = 2. If Bi = 2 (curve 2), then $\mathrm{Ma}_{q*} = 381.8$ at α = 2.86. Thus, at Bi = 0 there is no heat flow across the free boundary. The stability of the equilibrium state increases with an increase in heat transfer. The limiting case Bi → ∞ corresponds to the transition to an isothermal free surface, and no loss of stability occurs here. The critical Marangoni number approaches unity in this case. If We ≠ ∞, then there exists a value of $\alpha_*$ [$\alpha_* = 1.31$ for Bi = 2, We = $10^3$ (curve 3)] for which the denominator in (2.1) vanishes and the curve $\mathrm{Ma}_q(\alpha)$ becomes discontinuous. The region of stability decomposes into two parts. At $\alpha < \alpha_*$, it is located above curve 3. At $\alpha > \alpha_*$, it is bounded above by curve 3 and on the left by the straight line $\alpha = \alpha_*$. The left side of the neutral curve has the maximum $\mathrm{Ma}_q = 840$ at α = 0, while the right side reaches the minimum value $\mathrm{Ma}_{q*} = 381.2$ at α = 2.85. A decrease in We leads to a decrease in stability: the point of discontinuity $\alpha_*$ is shifted to the right and $\mathrm{Ma}_{q*}$ decreases. For all We ≠ ∞ $\mathrm{Ma}_q(1) = 0$, and at α → ∞ the curves $\mathrm{Ma}_q(\alpha, \mathrm{We})$ asymptotically approach the curve for an infinite Weber number. An increase in d is accompanied by an increase in the stability reserve: for d = 0.3, $\mathrm{Ma}_{q*} = 790.4$ at α = 3.68, while $\mathrm{Ma}_{q*} = 2014.9$ at α = 5.1 for d =

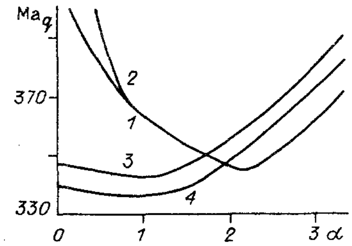


Fig. 3

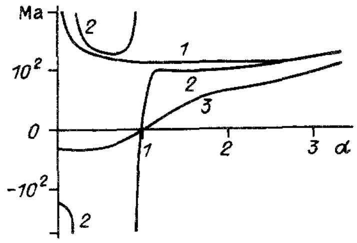


Fig. 4

0.5. This change can be attributed to the stabilizing effect of viscous forces near the solid surface.

Let us examine azimuthal perturbations. Figure 2 shows the dependence of $Ma_q$ on $\alpha$ for $d = 0.1$ and $Bi = 0$. For the undeformed free surface, curve 1 ($m = 1$) has the minimum $Ma_{q*} = 155.3$ at $\alpha = 1.21$. Curve 2 ($m = 2$) takes the minimum value $Ma_{q*} = 172$ at $\alpha = 0$ and then monotonically rises. Thus, for $Bi = 0$ and $We = \infty$, perturbations with $m = 1$ are more dangerous than perturbations with $m = 2$. The curves $Ma_q(\alpha, We)$ lie above curve 1 for $m = 1$, especially in the region of small $\alpha$. This is illustrated by curve 3, plotted for $We = 10^5$. Meanwhile, the two curves nearly merge at $\alpha > 0.4$. Thus, in the case of azimuthal perturbations ($m = 1$), allowing for the deformation of the free boundary leads to an increase in stability. Since stability decreases with We for $m = 2$, for $Bi = 0$ there also exists a value of We at which perturbations with $m = 2$ are more dangerous than perturbations with $m = 1$. As can be seen from Fig. 2, curve 4 ($m = 2$, $We = 10^2$) has the minimum $Ma_{q*} = 142.8$ at $\alpha = 0$ and lies below curve 1. If heat flows across the free surface ($Bi \neq 0$), then the mechanism by which the changeover to the most dangerous azimuthal modes occurs will be different. Figure 3 shows the graphs of $Ma_q(\alpha)$ for $d = 0.1$ and $Bi = 2$. For $We = \infty$, the minimum of curve 3 ($m = 2$) $Ma_{q*} = 342$ at $\alpha = 1.11$ lies below the minimum of curve 1 ($m = 1$) $Ma_{q*} = 344.3$ at $\alpha = 216$. There exists a value of $\alpha_*$ (for $Bi = 2$, $\alpha_* = 1.77$) such that at $\alpha > \alpha_*$ curve 3 lies above curve 1. Comparing curve 2, plotted for $We = 10^5$, and curve 1, we see that with allowance for the deformation of the free surface, the stability of the equilibrium state relative to azimuthal disturbances $m = 1$ increases. For $m = 2$, stability decreases with a decrease in We. Thus, $Ma_{q*} = 335.8$ (curve 4, $We = 10^3$, $Bi = 2$) at $\alpha = 1.06$. The case of a nondeformable free boundary will also be the most stable for subsequent azimuthal modes. Here, a decrease in We will have very little effect on the behavior of the neutral curve. Moreover, the stability reserve is larger for these perturbations than for the first two azimuthal modes. For example, for $d = 0.1$, $Bi = 2$, and $m = 3$, $Ma_{q*} = 415.9$ at $\alpha = 0$. The values of $Ma_{q*}$ increase with an increase in m.

Thus, perturbations with $m = 1$ will be the most dangerous at $Bi = 0$ for a nondeformable free surface. In the case when allowance is made for the deformation of the free boundary, there will also be a critical value of the Weber number $We_*$ such that perturbations with $m = 2$ will become the most dangerous for $We < We_*$. At $Bi \neq 0$, the increase in the wave numbers will become the deciding factor. At values of $\alpha$ less than a certain critical value $\alpha_*$, the azimuthal mode $m = 2$ will be the most dangerous. At $\alpha > \alpha_*$, $m = 1$ will present the greatest danger.

Comparison of our results with those obtained in [4] in the case of a thermally insulated solid cylinder showed that an equilibrium state with the temperature distribution (1.2) is more stable, since — in the case of ideal conduction — heat is removed from the solid surface.

4. Let us examine the stability of the equilibrium state of a cylindrical layer of fluid heated laterally by an internal solid cylinder ($Ma_q = 0$). Figure 4 shows graphs of the critical Marangoni numbers as a function of the wave numbers constructed from (2.1) for $Bi = 2$ and $d = 0.1$. Curve 1, corresponding to the case of a nondeformable surface, has the minimum $Ma_* = 114.5$ at $\alpha = 2.4$ (compare with $Ma_* = 50.3$ at $\alpha = 1.73$ for $Bi = 0$). In the case $We \neq \infty$, the denominator in (2.1) may vanish twice in relation to We. As is shown by curve 2, for $We = 10^4$ the points of discontinuity will be $\alpha_* = 0.195$ and $0.975$. Here, the graph of $Ma(\alpha)$ decomposes into three parts. The first part lies below zero. The second has the min-

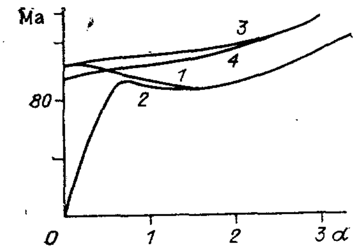


Fig. 5

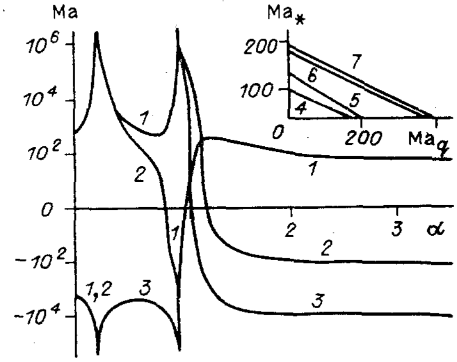


Fig. 6

imum Ma = 391.8 at α = 0.87. The third, increasing monotonically, reaches the local maximum Ma = 177 at α = 1.13 and then nearly merges with curve 1. A decrease in We is accompanied by a rightward shift of the first discontinuity and a leftward shift of the second. In this case, there is a value of $We_*$ ($We_* = 1.5 \cdot 10^3$ at Bi = 2, d = 0.1) for which the discontinuities disappear. This is illustrated by curve 3, plotted for We = $10^2$. For finite We, Ma(1) = 0, and at α → ∞ the curves Ma(α, We) asymptotically approach the curve for We = ∞.

Let us proceed to the description of azimuthal perturbations. The characteristic behavior of the neutral curves is shown in Fig. 5 for d = 0.1 and Bi = 2. Curves 1 and 3 correspond to the case of a nondeformable free surface. Here, curve 1 (m = 1) begins with Ma = 102 at α = 0 and has the minimum Ma = 88.2 at α = 1.41; for curve 3 (m = 2), the minimum value $Ma_* = 101.9$ is reached at α = 0. Stability decreases when allowance is made for the deformation of the free boundary, especially in the region of small wave numbers. Thus, for m = 1 and We = $10^4$ (curve 2), $Ma_* = 0$ at α = 0. For m = 2, the effect of deformation of the surface is not as substantial. For example, for We = $10^3$ (curve 4) the neutral curve again takes the minimum value at α = 0 ($Ma_* = 96.8$). Thus, azimuthal perturbations with m = 1 are the most dangerous for the equilibrium state of a cylindrical fluid layer heated by an internal solid cylinder.

The same problem was studied in [3]. Here, the free surface was assumed to have been nondeformable. The cumbersome nature of the formulas in [3] prevent us from comparing the results in general form. We therefore made a numerical comparison of the graphs of Ma(α) for axisymmetric perturbations. The calculations were performed with Bi = 0 and the results agreed quite well.

We also compared the neutral curves $Ma_q = 0$ with the curves for Ma = 0 obtained in [4] in the case of a thermally insulated solid surface. The comparison showed that the equilibrium state of a cylindrical fluid layer heated by internal sources is more stable than in the case of heating of the solid surface. The same result was observed for a plane layer in [4].

5. Let us examine the effect of internal heat sources on the stability of equilibrium with heating of the solid surface. Figure 6 presents graphs of the dependence of Ma on α with d = 0.1, Bi = 2, and We = $10^4$. The graphs were plotted from (2.1) for $Ma_q = 10^2, 10^3$, and $10^4$ (curves 1-3). Here, as in the case $Ma_q = 0$, the denominator vanishes twice in relation to We. Figure 6 also shows the dependence of $Ma_* = \min_\alpha Ma(\alpha)$ on $Ma_q$ at d = 0.1 (curve 4) and 0.3 (curve 6) for Bi = 2, We = ∞. At $Ma_q = 0$, we obtain critical Marangoni numbers determining the boundary of stability in the absence of internal heat sources ($Ma_* = 114.5$ for d = 0.1 and 190.7 for 0.3). The quantity $Ma_*$ vanishes at $Ma_q = 380$ for d = 0.1 and at $Ma_q = 790$ for d = 0.3. For comparison, we show the dependence of $Ma_*$ on $Ma_q$ with m = 1 and the same values for the other parameters. In the absence of internal heat sources ($Ma_q = 0$) in the case of azimuthal perturbations, $Ma_* = 88.2$ at d = 0.1 (curve 5) and 177.6 at d = 0.3 (curve 7). Accordingly, $Ma_*$ vanishes at $Ma_q = 344$, d = 0.1 and at $Ma_q = 770$, d = 0.3.

Thus, azimuthal perturbations (m = 1) are the most dangerous for the equilibrium state of a cylindrical layer of fluid (1.1). Here, the effect of these disturbances weakens with an increase in d (at roughly d > 0.5) and the neutral curves associated with axisymmetric and azimuthal perturbations nearly merge.

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# NUMERICAL STUDY OF STEADY-STATE REGIMES OF ROTATIONAL–GRAVITATIONAL CONVECTION

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This article examines the problem of the two-dimensional convection of a viscous incompressible fluid in a rotating horizontal layer with free isothermal boundaries. Approximate solutions are sought by the Galerkin method. We numerically study stability and bifurcative steady-state solutions with a change in the Rayleigh number. The Galerkin method was used in [1-3] to perform calculations for the same problem (also see [4, 5]). In the present investigation, we study transitions in the class of steady-state solutions and calculate the corresponding bifurcative values of R.

Results are presented for a Galerkin system of 62 equations. Equilibria are determined by Newton's method with continuation with respect to the parameter R. We find bifurcative values of R corresponding either to the generation of a pair of equilibria or a shift in the type of stability of the equilibrium. Using the results in [6], we fix the remaining parameters (Prandtl and Taylor numbers, wave number) so that the loss of stability of relative mechanical equilibrium with an increase in R is monotonic. Here, secondary steady-state solutions branch into the subcritical region and are unstable.

Nonetheless, we observed several branches of stable steady motion. These branches appear by different methods with a monotonic increase in R. Of particular interest is the following mechanism: the generation of a pair of unstable equilibria "from air" and their return to stability as a result of Andronov–Hopf bifurcation.

1. Let a viscous heat-conducting fluid fill a horizontal layer of thickness H with nondeformable free boundaries. The temperatures on the lower and upper boundaries of the layer are $T_1$ and $T_2$, respectively. In the main regime, the fluid rotates as a rigid body with the angular velocity $\Omega$ around the vertical axis. The motion of the fluid is described by the equations of free convection in the Oberbeck–Boussinesq approximation [7, 4]. We will ignore the centrifugal force.

In a cartesian coordinate system (x, y, z) rotating together with the field, the fields of relative velocity $v = (v_1, v_2, v_3)$ and temperature are assumed to be independent of the coordinate y. We introduce the stream function $\psi$: $v_1 = \partial\psi/\partial z$, $v_3 = -\partial\psi/\partial x$. The equations of motion have the following dimensionless form:

$$\partial\Delta\psi/\partial t = J(\psi,\ \Delta\psi) + \Delta^2\psi + \tau\partial v/\partial z - G\partial T/\partial x,$$
$$\partial v/\partial t = J(\psi,\ v) + \Delta v - \partial\psi/\partial z,\ \partial T/\partial t = J(\psi,\ T) + \mathrm{Pr}^{-1}\Delta T - \partial\psi/\partial x. \tag{1.1}$$

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